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SEMI-ACTIVE HYDRO-PNEUMATIC SUSPENSION ACCURATE LINEARIZATION LQG CONTROL BASED ON AHP

Zhaolong Cao

Department of Fundamental Science Teaching, Yancheng Institute of Technology, Yancheng 224001, P. R. China

Abstract

In order to improve the ride comfort and safety of engineering vehicle, the 1/4 vehicle semi-active hydro-pneumatic suspension dynamic model was established, and analysed the nonlinear characteristics of the hydro-pneumatic suspension spring force and damping force. For the nonlinear characteristics of the system, the suspension system nonlinear model accurate linearization was realized by the accurate linearization method based on the differential geometry theory. The LQG controller that applied to linear system was designed by applying the optimal control theory, and the weighted coefficient of LQG control performance index was determined by analytic hierarchy process (AHP). At last, the system control model was simulated in MATLAB/Simulink. The results show that: the ride comfort can be improved significantly by semi-active hydro-pneumatic suspension than passive hydro-pneumatic suspension, and the handling stability and running safety can be also improved.

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*Corresponding author.

E-mail address: long_066@126.com (Zhaolong Cao).

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1. Introduction

The engineering vehicles are always working under complex road conditions, which requires high performance of suspension system. The hydro-pneumatic suspension can transfer pressure by oil and take the gas as the elastic medium, so the suspension has the characteristics of variable stiffness and variable damping. It can meet the requirement of handling stability and running safety, and now has been applied to most engineering vehicles [1].

High pressure pipeline of semi-active hydro-pneumatic suspension was used to connect with the electromagnetic proportional control valve in external of hydro-pneumatic spring, so the suspension can output adjustable damping force by adjusting the valve orifice area [2]. So study on the semi-active control method of hydro-pneumatic suspension can give full play to the vehicle performance and improve the working efficiency. But the stiffness and damping of the suspension has very obvious nonlinear features [1-3]. For the nonlinear system, the linearization processing to it is the more effective way, such as the Taylor series expansion method and differential geometry method. Taylor series expansion method is a classical linear method, the model can be expanded near the equilibrium point, the high-order was abandoned, obtain the larger error linear approximation. The accuracy linearization method that developed on the basis of the differential geometry, to make the nonlinear system through a state transformation and feedback, so the accuracy linearization of all or part dynamic characteristics of the system can be realized, the complex nonlinear system problem was transformed into linear system problem [4, 5]. Because of the extensive application of control theory in the vehicle, the vehicle performance is improved greatly, such as the LQG control, which has been developed relatively mature in linear control field, through determine the index weighting coefficient, system state variables and control variables weighted matrix, provides some design space for the designer [6]. But the determination of index weighting coefficient always depend on trial method or designer's experience [7-9], can not meet the requirements of engineering [10, 11]. Studied the determination of LQG controller weight coefficient by the analytic hierarchy process, realized the weighted coefficient online control, it relatively has universality. Because of the complex of the road has high requirements of suspension safety performance and its controller, and AHP can meet the design requirements by the weighted coefficient online control according to the importance of indicators.

Therefore, the accuracy linearization method was selected to realize linearization of the nonlinear model, and the LQG controller was designed for linear system in this paper. The body acceleration, suspension dynamic travel, tire dynamic displacement, suspension power consumption, and control energy as the performance evaluation index of LQG controller, and AHP was used to determine the weighting coefficient of LQG controller.

2. Characteristic Analysis of Hydro-Pneumatic Suspension

2.1. System dynamic model

The mono-tube hydro-pneumatic suspension was used in this paper, the model of the suspension is shown in Figure 1. In the Figure 1, m is the sprung mess; m_w is the unsprung mass; K_t is the stiffness of tire; z_s is the displacement of sprung mess; z_w is the displacement of unsprung mess; and z_g is road input displacement.

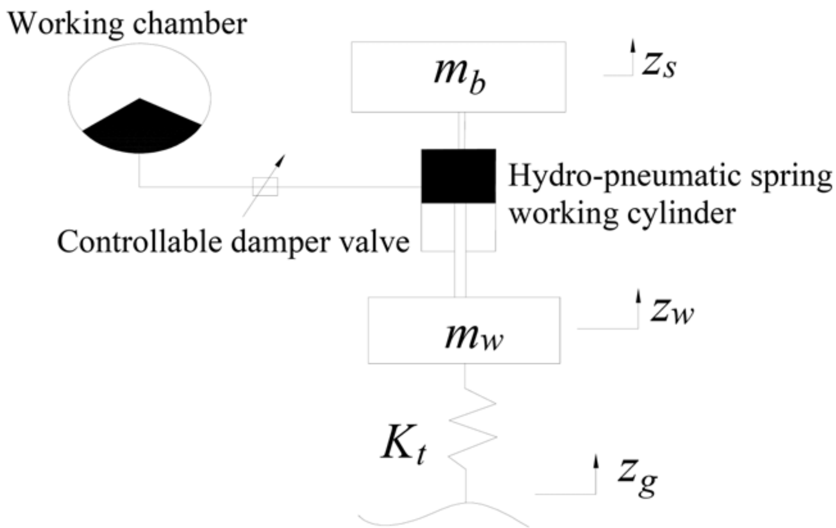


Figure 1. 1/4 vehicle model of hydro-pneumatic suspension.

And the dynamic differential equation of system can be established as follow:

$$\begin{cases} m_b\ddot{z}_s = -K_s(z_s - z_w) - C_s(\dot{z}_s - \dot{z}_w) - \Delta F, \\ m_w\ddot{z}_w = K_s(z_s - z_w) + C_s(\dot{z}_s - \dot{z}_w) \\ \qquad\qquad - K_t(z_w - z_g) + \Delta F, \end{cases} \tag{1}$$

where K_s is the nonlinear stiffness of hydro-pneumatic suspension; C_s is the nonlinear damping; $C_s(\dot{z}_s - \dot{z}_w)$ is uncontrollable damping force; ΔF is the damping force produced when the oil flows through the damping valve, it is the controllable damping force, and the condition of Equation (2) should be met

$$\Delta F = \begin{cases} \Delta F & \Delta F(z_s - z_w) > 0, \\ 0 & \Delta F(z_s - z_w) \le 0. \end{cases} \tag{2}$$

The state vector can be defined as

$$\boldsymbol{Y} = \begin{bmatrix}\ddot{z}_s & z_s - z_w & z_w - z_g & \dot{z}_s - \dot{z}_w\end{bmatrix}^T$$

$$= [y_1 \;\; y_2 \;\; y_3 \;\; y_4]^T.$$

So the general form of semi-active hydro-pneumatic suspension nonlinear system can be expressed as Equation (3)

$$\begin{cases}\dot{\boldsymbol{X}} = \boldsymbol{F}(\boldsymbol{X}) + \boldsymbol{G}(\boldsymbol{X})u + \eta w, \\ \boldsymbol{Y} = \boldsymbol{h}(\boldsymbol{X}) + \boldsymbol{d}(\boldsymbol{X})u,\end{cases} \tag{3}$$

where $u = \Delta F$; $\eta = [0 \quad 0 \quad 0 \quad -1]^T$; $w = \dot{z}_g$;

$$\boldsymbol{G}(\boldsymbol{X}) = [-1/m_b \quad 1/m_w \quad 0 \quad 0]^T;$$

$$\boldsymbol{d}(\boldsymbol{X}) = [-1/m_b \quad 0 \quad 0 \quad 0]^T;$$

$$\boldsymbol{F}(\boldsymbol{X}) = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} = \begin{bmatrix} -K_s x_3 / m_b - C_s(x_1 - x_2)/m_b \\ F_s x_3 / m_w + C_s(x_1 - x_2)/m_w - K_t x_4 / m_w \\ x_1 - x_2 \\ x_2 \end{bmatrix};$$

$$\boldsymbol{h}(\boldsymbol{X}) = \begin{bmatrix} -K_s x_3 / m_b - C_s(x_1 - x_2)/m_b \\ x_3 \\ x_4 \\ x_1 - x_2 \end{bmatrix}.$$

2.2. Stiffness and damping model of hydro-pneumatic suspension

The stiffness of hydro-pneumatic suspension can be defined by the Equation (4) [5].

$$K_s = \left|\frac{dF_s}{dx}\right| = \frac{\gamma A P_p V_p^{\gamma}}{l[V_p + Ax_3 / l]^{\gamma+1}}, \tag{4}$$

where V_p is the volume of working chamber in the static equilibrium position; A is the effective area of the piston; l is the lever ratio of suspension; P_p is the pressure of working chamber in the static equilibrium position; and γ is the air polytropic exponent.

The damping of hydro-pneumatic suspension can be defined by the Equation (5)

$$C_s = \frac{dF_d}{d\dot{x}} = \frac{\rho A_3^3 \dot{x}_3 \operatorname{sgn} \dot{x}_3}{C_d^2[A_1 + A_2(1 - \operatorname{sgn} \dot{x}_3)/2]^2}, \tag{5}$$

where C_d is flow coefficient; A_1 is orifice area; A_2 is check valve flow area; A_3 is the area of outside oil chamber; ρ is fluid density; and $\operatorname{sgn} \dot{x}_3 = \begin{cases} 1 & \dot{x}_3 > 0 \\ -1 & \dot{x}_3 < 0 \end{cases}.$

From the Equations (4)-(5), we can see that the stiffness and damping of hydro-pneumatic suspension has obvious non-linear characteristics.

3. Accuracy Linearization of Hydro-Pneumatic Suspension

3.1. The differential geometry theory

The general form of single input and single output nonlinear system can be described by Equation (6)

$$\begin{cases} \dot{\boldsymbol{X}} = \boldsymbol{F}(\boldsymbol{X}) + \boldsymbol{G}(\boldsymbol{X})u, \\ \boldsymbol{Y} = \boldsymbol{h}(\boldsymbol{X}), \end{cases} \tag{6}$$

where $\boldsymbol{X} \in \boldsymbol{R}^n$ is the state variable; $\boldsymbol{F}$ and $\boldsymbol{G}$ are the smooth vector field of $\boldsymbol{R}$, with continuous derivatives of order infinity; $\boldsymbol{Y} \in \boldsymbol{R}$ is the output variable; $\boldsymbol{h}(\boldsymbol{X})$ is a sufficiently function; and $u \in \boldsymbol{R}$ is the input control variable.

For all the x in the neighbourhood of $x = x_0$, the Equation (7) can be established by an integer r, so the integer r is the system relative order. The Equation (7) can be as follow:

$$\begin{cases} L_G L_F^k \boldsymbol{h}(\boldsymbol{X}) = 0, & 0 \le k < r-1, \\ L_G L_F^{r-1} \boldsymbol{h}(\boldsymbol{X}) \ne 0, & \end{cases} \tag{7}$$

where L is Lie derivative functional operator, which can be described by Equation (8)

$$L_G F = \frac{\partial F}{\partial x} G. \tag{8}$$

When r is the relative order of Equation (6), the state feedback transformation can be described by Equation (9)

$$u = \frac{1}{L_G L_F^{r-1} \boldsymbol{h}(\boldsymbol{X})} \left[- L_F^r \boldsymbol{h}(\boldsymbol{X}) + v \right]. \tag{9}$$

3.2. Accuracy linearization of system model

According to Equations (7)-(9) to analysis each variable of original system.

Let $h_1(\boldsymbol{X}) = x_1$, when $k = 0$, then

$$L_F^0 h_1(\boldsymbol{X}) = h_1(\boldsymbol{X}) = x_1;$$

$$L_G L_F^0 h_1(\boldsymbol{X}) = \frac{\partial h_1(\boldsymbol{X})}{\partial \boldsymbol{X}} \boldsymbol{G}(\boldsymbol{X}) = -\frac{1}{m_b}.$$

So the relative order is 1.

$$L_F^1 h_1(\boldsymbol{X}) = \frac{\partial h_1(\boldsymbol{X})}{\partial \boldsymbol{X}} \boldsymbol{F}(\boldsymbol{X}) = \frac{-K_s x_3}{m_b} - \frac{C_s(x_1 - x_2)}{m_b};$$

$$u_1 = \frac{-L_F^1 h_1(\boldsymbol{X}) + v}{L_G L_F^0 h_1(\boldsymbol{X})} = -K_s x_3 - C_s(x_1 - x_2) - m_b v_1.$$

Similarity, we can get

When $h_2(\boldsymbol{X}) = x_2$, the relative order is 1,

$$u_2 = -K_s x_3 - C_s(x_1 - x_2) - K_t x_4 - m_w v_2.$$

When $h_3(\boldsymbol{X}) = x_3$, the relative order is 2,

$$u_3 = -K_s x_3 - C_s(x_1 - x_2) + \frac{m_b K_t x_4}{m_b + m_w} - \frac{m_b m_w}{m_b + m_w} v_3.$$

When $h_4(\boldsymbol{X}) = x_4$, the relative order is 2,

$$u_4 = -K_s x_3 - C_s(x_1 - x_2) - K_t x_3 - m_w v_4.$$

Through comprehensive analysis, the system semi-active control rate can be described by Equation (10)

$$u = -K_s x_3 - C_s(x_1 - x_2) - K_t x_3 - v. \tag{10}$$

The Equation (10) put in Equation (3), we can get the output equation of system state equation

$$\begin{cases} \dot{\boldsymbol{X}} = \boldsymbol{AX} + \boldsymbol{B}v + \eta w, \\ \boldsymbol{Y} = \boldsymbol{CX} + \boldsymbol{D}v, \end{cases} \tag{11}$$

where A is state matrix, B and η are input matrices, C is output matrix and D is transfer matrix.

4. System Controller Design

4.1. LQG controller design

The integral value of weighted square sum of body acceleration, suspension dynamic travel, tire dynamic displacement, suspension power consumption, and control energy as the target performance index J of LQG controller. The expression can be described by Equation (12)

$$J = \lim_{T\to\infty} \frac{1}{T} \int_0^T \begin{bmatrix} q_1 \dot{x}_1^2 + q_2 x_3{}^2 + q_3 x_4{}^2 \\ + q_4(x_1 - x_2)^2 + \rho v^2 \end{bmatrix} dt, \tag{12}$$

where q_1, q_2, q_3, q_4, and ρ are the weighting coefficient of performance index, respectively.

Equation (12) can be transformed into the form of Equation (13)

$$\boldsymbol{J} = \lim_{T\to\infty} \frac{1}{T}\int_0^T \left[\boldsymbol{Y}^T \mathcal{Q}_0 \boldsymbol{Y} + \boldsymbol{\nu}^T \ \rho\boldsymbol{\nu}\right] dt, \tag{13}$$

where $\mathcal{Q}_0 = \text{diag}[q_1 \ q_2 \ q_3 \ q_4]$; $\rho = [\rho]$.

The equation $\boldsymbol{Y} = \boldsymbol{CX} + \boldsymbol{D}\nu$ put in Equation (13), we can get Equation (14)

$$\boldsymbol{J} = \lim_{T\to\infty} \frac{1}{T}\int_0^T \left[\boldsymbol{X}^T \ \mathcal{Q}\boldsymbol{X} + \boldsymbol{\nu}^T \boldsymbol{R\nu} + 2\boldsymbol{X}^T \ \boldsymbol{N\nu}\right] dt, \tag{14}$$

where $\mathcal{Q} = \boldsymbol{C}^T \mathcal{Q}_0 \boldsymbol{C}$ is the weighting matrix of state variables; $\boldsymbol{R} = \rho + \boldsymbol{D}^T \mathcal{Q}_0 \boldsymbol{D}$ is the weighting matrix of control variables; and $\boldsymbol{N} = \boldsymbol{C}^T \mathcal{Q}_0 \boldsymbol{D}$ is the weight of cross terms.

From the Riccati equation (15), we can get $\boldsymbol{P}$

$$\boldsymbol{PA} + \boldsymbol{A}^T \boldsymbol{P} - (\boldsymbol{PB} + \boldsymbol{N})\boldsymbol{R}^{-1}(\boldsymbol{B}^T \boldsymbol{P} + \boldsymbol{N}^T) + \mathcal{Q} = 0. \tag{15}$$

Then $\boldsymbol{\nu} = -\boldsymbol{R}^{-1}(\boldsymbol{B}^T \boldsymbol{P} + \boldsymbol{N}^T)\boldsymbol{X} = \boldsymbol{KX}$ is the semi-active optimal control rate, put in Equation (10), we can get the final nonlinear state feedback. The diagram of control system is shown in Figure 2.

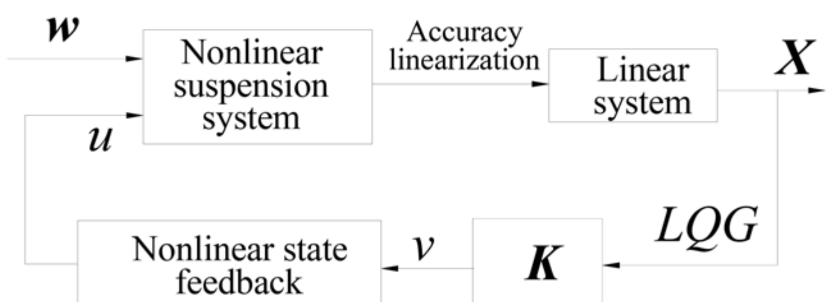


Figure 2. The diagram of control system.

4.2. The determination of LQG controller weighted coefficient

Analytic hierarchy process is a method of multi-objective programming and decision making, we can use it to determine the subjective weights of each evaluation index [10, 11].

4.2.1. Determination steps of weighted coefficient by AHP

(a) *The construction of evaluation index comparison matrix* $\boldsymbol{H}$

Let h_{ij} as the importance comparison value of index i and j, the comparison table of the relative importance of each index is shown in Table 1.

Table 1. The comparison value of index importance

Index i/j	Equal important	Slightly important	Most important	Important	Very important
h_{ij}	1	3	5	7	9

If the two index relative importance between the two comparison value, then the value 2, 4, 6, 8 should be taken. So, we can get the subjective weighted comparison matrix $\boldsymbol{H}$ by the Table 1.

(b) *Calculate the multiply vector* $\boldsymbol{M}$

The elements of $\boldsymbol{M}$ are: $M_i = \prod_{j=1}^{n} h_{ij}$.

(c) *Calculate the vector* $\overline{\boldsymbol{V}}$

The elements of $\overline{\boldsymbol{V}}$ are: $\overline{\boldsymbol{V}}_i = M_i^{1/n}$.

(d) *Calculate the regular vector* $\boldsymbol{V}$ *of vector* $\overline{\boldsymbol{V}}$

The regular vector $\boldsymbol{V}$ can be described by the Equation (16). The elements of $\boldsymbol{V}$ are the weighted coefficient of each evaluation index.

$$\boldsymbol{V} = \overline{\boldsymbol{V}} \Big/ \sum_{i=1}^{n} \overline{\boldsymbol{V}}_i. \tag{16}$$

(e) *Check the consistency of the comparison matrix*

The consistency of the comparison matrix should be checked by the Equation (17)

$$R_c = (\lambda_{\max} - n) / R_I(n-1), \tag{17}$$

where $\lambda_{\max} = \sum_{i=1}^{n} [(\boldsymbol{HV})_i / nV_i]$; R_I is the random consistency index of comparison matrix $\boldsymbol{H}$, and the value of R_I is shown in Table 2. When $n = 5$, $R_I = 1.2$.

Table 2. The value of random consistency index R_I

n	2	3	4	5	6	7	8	9
R_I	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45

When $R_C < 0.1$, through the test of consistency, otherwise consistent correction of matrix $\boldsymbol{H}$ should be carried out by the method in reference [12].

4.2.2. The finally determination of weighted coefficient

Because of the big difference between the unit and the magnitude order of each evaluation index, when compare its relativity, the same scale quantitative proportion coefficient of the evaluation index should be obtained by the passive suspension [11]. So, the performance standard deviation σ_i in a particular condition of passive suspension can be chosen. Suppose that the same scale quantitative proportion coefficient of body vertical acceleration standard deviation σ_1 is 1, then the same scale quantitative proportion coefficient β_i of another performance index can be determined by Equation (18)

$$\sigma_1^2 \times 1 = \sigma_i^2 \times \beta_i, \quad i = 1, 2, \ldots, n. \tag{18}$$

Let the subjective weighted proportion coefficient of body vertical acceleration is 1, then the subjective weighted proportion coefficient γ_i and the final weighted proportion coefficient q_i of another performance index can be determined by Equation (19)

$$\begin{cases} V_2 = V_i / \gamma_i \\ q_i = \beta_i \times \gamma_i \end{cases}, \; i = 1, 2, \ldots, n. \tag{19}$$

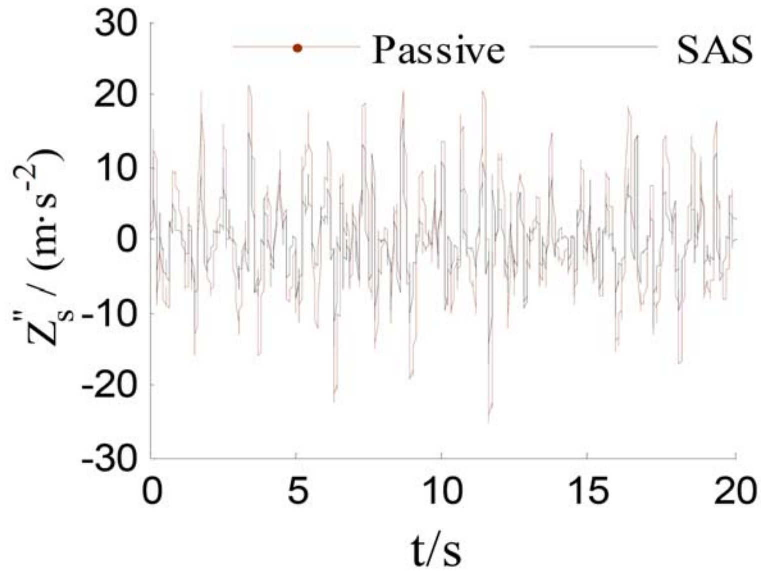
5. Simulation Analysis of System

The first-order filtering white noise can be used as the road input

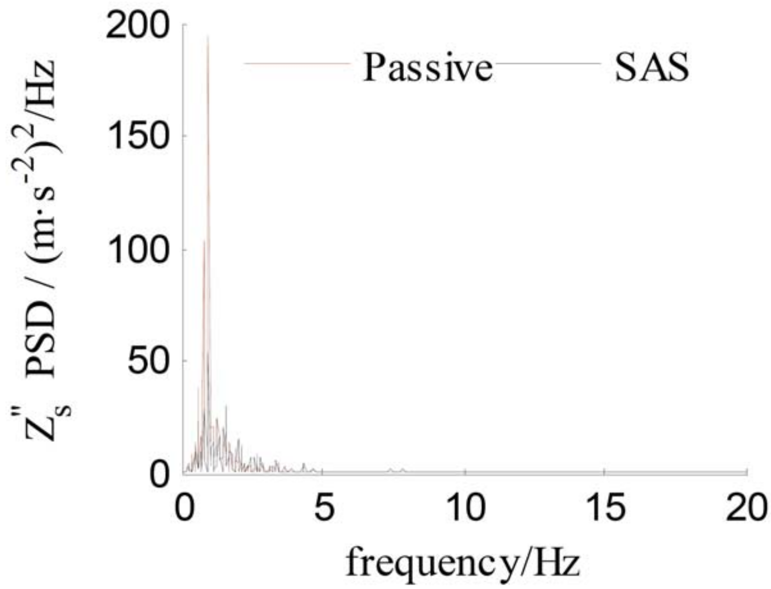
$$\dot{z}_g(t) = -2\pi f_0 z_g(t) + 2\pi\sqrt{G_0 u_c}\, w(t), \tag{20}$$

where f_0/Hz is the lower cut-off frequency; $G_0/(\mathrm{m}^3 \cdot \mathrm{cycle}^{-1})$ is the road roughness coefficient; u_c is the vehicle speed; w is the Gauss white noise distribution, the mean value is 0 and the noise intensity is 1.

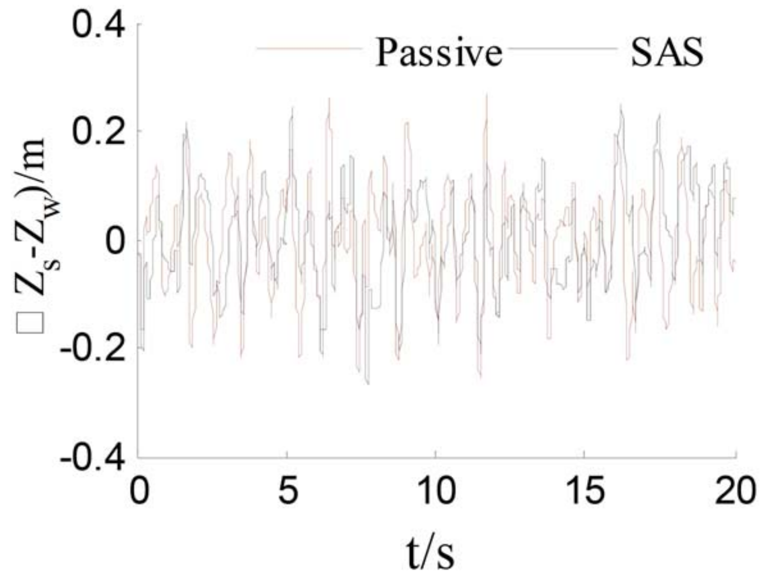
According to the complex of the working road, we define $u_c = 10\mathrm{m/s}$, $G_0 = 256 \times 10^{-6}\mathrm{m}^3/\mathrm{cycle}$, and $f_0 = 0.1\mathrm{Hz}$. Through the simulation analysis in Matlab/Simulink, the performance comparison of hydro-pneumatic suspension and passive suspension can be shown in Figure 3.



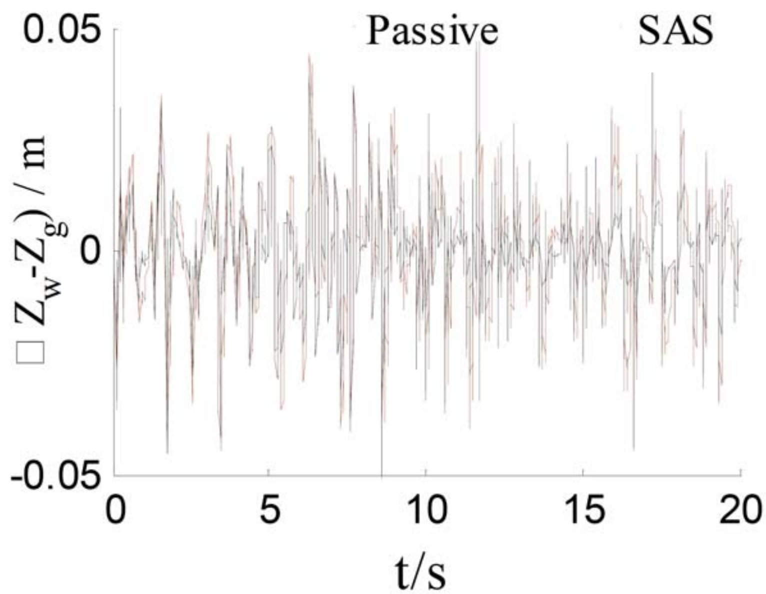
(a) Body Acceleration



(b) Body Acceleration PSD



(c) Suspension Displacement



(d) Tire Dynamic Displacement

Figure 3. The performance comparison of two suspension.

The comparison of each performance standard difference passive suspension and semi-active hydro-pneumatic suspension can be shown in Table 3.

Table 3. The comparison of each performance standard difference

Suspension type	$\ddot{z}_s/(\mathrm{m \cdot s^{-2}})$	$(z_s - z_w)/\mathrm{m}$	$(z_w - z_g)/\mathrm{cm}$
Passive suspension	8.243	0.105	1.54
SAS	4.734	0.082	1.12

From Table 3, we can find that the body acceleration, suspension displacement, and tire dynamic displacement of semi-active suspension are reduced 42.6%, 21.9%, and 27.3%. The performance of suspension has been improved a lot, and it can meet the requirement of engineering.

6. Conclusion

In this paper, the nonlinear characteristics of hydro-pneumatic suspension spring force and damping force has been analyzed. The state feedback accuracy linearization nonlinear transformation of engineering vehicle semi-active hydro-pneumatic suspension has been realized by accuracy linearization method based on differential geometry theory. Optimal control rate of linear system can be got by LQG control theory, and then the whole nonlinear state feedback of nonlinear systems can be got. According to the requirements of ride comfort and safety, the LQG controller performance index weighted coefficients are determined by the analytic hierarchy process, the on-line control of weighted coefficients are realized, with the more universal and practical.

Through the simulation analysis, the body vertical acceleration can be reduced greatly, improved the ride comfort. And the comprehensive performance of the engineer vehicle can also be improved, can better meet the security requirement.

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